ESTIMATING THE COBB DOUGLAS PRODUCTION FUNCTION INCLUDING THE EXPORT AND OPENNESS IN THE CASE OF ROMANIA

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Abstract: Economic convergence theories are closely related to economic growth theories. The first to study the economic growth phenomenon were the classics A. Smith, D. Ricardo, Th. Malthus, whose models of so-called classic models, do not take into account the contribution of technical progress in increasing production per capita. In order to analyze the convergence process, as a result of economic growth, a series of studies have been created to check the convergent or divergent nature of economies. Thus, in order to identify present sources of economic growth for Romania in our study we have used the Cobb-Douglas type production function. The variables that are the base of this model are represented by work factors and capital stock, to which we have added two explicative variables of economic growth: export and the openness degree of the economy. The two economic growth variables have been included in the model due to their favorable influence on the Solow residue. To estimate this production function, quarterly statistical data from the period between 2000 – first quarter and 2014 – fourth quarter have been used; the source of the data was Eurostat. As to what the first estimated model is concerned, the Cobb-Douglas production function including the export variable are both valid in Romania’s case, this have the parameters of the exogenous variables significantly different from zero, while the second estimated model, which contains the openness variable, is not valid. Its independent variable coefficient is not significantly different from zero, at the level of the entire population. This shows us that the inclusion of the degree of openness of the economy variable in the model affects the significance degree of the model and in order to validate it, the variable must be eliminated. Therefore, we can state that in Romania an increase of the openness degree of the economy due to capital imports would not generate an improvement in what type of technologies are used.

Keywords: economic growth, export, openness, econometric model.

JEL classification: F43, Q56, B23.

Introduction
The economic growth process is one of the most talked about aspects in specialized literature, due to the major impact it has on the population of the country. This is the reason for which the making or maintaining a high growth rhythm is a major objective of macroeconomic politics. The preoccupation for studying the economic growth process has been around beginning with the representatives of the classical school, continuing with Keynsists, Neokeysists and neoclassicist. A series of empirical studies have tested the correlation between the dynamics in different factors and in the economic growth process. Thus, in order to identify present growth sources of the economy for Romania, we have used a Cobb-Douglas type production function.

1. Data analysis
In this paper, the explanatory variables of the model are represented by work factors and capital stock, to which we have added the export and the openness of the economy. The inclusion of the openness and the exports in this model is caused by the favorable
influence which they have on the Solow residue. As such, an increase of exports will determine a growth in the efficiency of internal companies, while the increase of the openness will lead to generating an improvement of the technologies in use, following the imports of capital.

In order to estimate the production function, considering the two variables export and openness, we have used the statistical quarterly data, the period we studied being the first quarter of 2000 (Q1) until the 4th quarter of 2014 (Q4), for which there is available data. The quarterly information provided by Eurostat regarding the gross forming of the fixed capital and the variations of stock have helped us determine the gross investment as being the sum of the two. The capital stock and export are expressed in millions of euros, at the price levels of 1999, and the openness degree of the economy (calculated by reporting the sum of exports and imports to the GDP) is expressed in percentages (Marinaș, 2008: 229). All the four data series have been deseasonalized and turned into logarithms.

Initially we estimated the production functions as having the capital stock, working population and export, after which we added a new explanatory variables of economic growth—the openness.

2. The Cobb-Douglas production function including the export variable

In order to estimate the production function by including the export variable, we start from this equation:

\[ GDP = A \times K^\alpha \times L^\beta \times \text{Exp}^\gamma \times \varepsilon_t \]  

(1)

where:

\( \gamma \) is the elasticity of the GDP based on the evolution of exports.

Following the transformation into a logarithm of the new production function, we get:

\[ \log(GDP) = \log(A) + \alpha \times \log(K) + \beta \times \log(L) + \gamma \times \log(\text{Exp}) + \varepsilon \]  

(2)

The errors that result from the estimation of equation 2 were positively correlated (DWcalc = 0.853213). Thus, in the case of this model, in order to eliminate positive self-correlation, we will use the Hildreth-Lu algorithm. This supposes estimating the parameters of the model in which the \( \rho \) parameter has an unknown value. The \( \rho \) parameter will be estimated in an algorithmic manner (Stancu, 2011: 249).

With the help of this algorithm, we estimate the linear regression linear log model’s parameters 2). The first step of this algorithm is determining the type of positive self-correlation (\( \rho > 0 \)) or negative (\( \rho < 0 \)), with the help of the Durbin-Watson statistic, in order to find the positive value interval (Andrei et al, 2008: 126): either [0, 1] or [-1, 0]. In our case, the self-correlation of errors is positive, because \( 0 < \text{DWcalc} = 0.853213 < d_1 = 1.48 \).

The second step is fixing the values of the \( \rho \) parameter, which will be used in estimating the parameters of the model:

\[ \log(GDP_t) - \rho \log(GDP_{t-1}) = \log(A) (1 - \rho) + \alpha (\log(K_t) - \rho \log(K_{t-1})) + \beta (\log(L_t) - \rho \log(L_{t-1})) + \gamma (\log(\text{Exp}_t) - \rho \log(\text{Exp}_{t-1})) + \varepsilon_t \] 

(3)

For each fixed value for the \( \rho \) parameter, by using the method of the smallest squares, we estimate the parameters of the regression model given by the above relation, thus obtaining the \( \alpha, \beta \) and \( \gamma \) estimators. Also, in the case of each step, we obtain the series of residue and the sum of the squares of the residue. The last step of this algorithm is eliminating the parameters of the relation that were obtained before (\( \alpha, \beta \) and \( \gamma \), for which the sum of the squares of the residue is minimal and which are the same as those of the initial equation.
In our case, after completing the above steps, we have found that the value of the parameter $\rho$ is between the 0.90-0.95 interval, more exactly, the value that checks the last step of the algorithm is $\rho = 0.94$. Next, we will present the model that shows the relation between the GDP, capital stock, working population and export, after we have eliminated the self-correlation of errors.

Table 1: Estimating the parameters of the regression model (3)

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C(1)$</td>
<td>4.177032</td>
<td>1.339827</td>
<td>3.117592</td>
</tr>
<tr>
<td>$C(2) = \alpha$</td>
<td>0.193660</td>
<td>0.067327</td>
<td>2.876383</td>
</tr>
<tr>
<td>$C(3) = \beta$</td>
<td>-0.024986</td>
<td>0.119526</td>
<td>-0.209045</td>
</tr>
<tr>
<td>$C(4) = \gamma$</td>
<td>0.487973</td>
<td>0.085637</td>
<td>5.698137</td>
</tr>
</tbody>
</table>

R-squared: 0.487162
Adjusted R-squared: 0.459189
S.D. dependent var: 0.043760
S.E. of regression: 0.032181
Akaike info criterion: -3.969496
Schwarz criterion: -3.828646
F-statistic: 17.41546
Durbin-Watson stat: 1.689691

Source: the authors’ calculus

According to the data from the table above, we can write the following relation:

$$GDP = 0.62 \times K^{0.19} \times L^{-0.02} \times \text{Exp}^{0.48} \times u_t$$

(4)

The results show that all coefficients of the equation are significantly different from zero at the sample level, and the intensity of the link between the variables of the same level is average (Adjusted $R^2 = 0.459189$). The elasticity of the potential GDP based on the working population is reduced as compared to the initial production function, so an increase of the working population of 10% generated a decrease of 0.2% of the potential GDP by increasing the productivity of work. The increase of exports generated an improvement of work efficiency and capital stock in the exporting sectors of the economy. Given the value of the GDP elasticity, we can say that an increase of exports of 10% determined an average economic growth of 4.8% during 2000-2014. We can also observe that this factor had the highest contribution to the increase of the potential GDP, followed by the stock capital of the economy, a factor that influenced the potential GDP in the same manner, leading to an increase of capital stock of 10%, potential GDP grows with 0.19%. This value corresponds to some decreasing marginal efficiencies of the capital factor. Moreover, we can say that this production function is characterized by decreasing efficiencies, because the sum of the $\alpha$, $\beta$ and $\gamma$ factors is 0.65.

If we check the link between the four variables, at the level of the entire population, we can see that only the free term, meaning $\alpha$ and $\gamma$, are significantly different from zero; the $\beta$ parameter is not significantly different from zero, because the value of the Student statistic is smaller in absolute value ($t$-statistic $\beta = |-0.209045|$) than the table value equal to 2.00 for a $P=0.05$ probability and $df = 59$ degrees of freedom. In order for this model to become
valid, the “working population variable” needs to be eliminated from the model. Thus, after re-estimating the new model, we have obtained the following results:

Table 2: Estimating the new regression model

| Dependent Variable: LOG(GDP)-(0.94*LOG(GDP(-1))) | Method: Least Squares |
| Sample (adjusted): 2000Q2 – 2014Q4 |
| Included observation: 59 after adjustments |
| \( \log(GDP)^{(0.94*\log(GDP(-1)))}=C(1)^{(-1-0.94)}+C(2)^{*(\log(K-0.94*\log(K(-1)))}}+C(3)^{*(\log(Exp)-0.94*\log(Exp(-1)))} \) |

| Testing the significance of the parameters (Student Test) | LogA | 3.969735 (Prob.=0.0000) |
| α | 0.196879 (Prob.=0.0037) |
| γ (Exp) | 0.482715 (Prob.=0.0000) |
| Adjusted R-squared | 0.468424 |
| Independence of errors (Durbin Watson Test) | DW<sub>calc</sub> | 1.693219* |
| Homoscedasticity of errors (White Test) | F<sub>calc</sub> | 0.449339 (Prob.=0.8119) |
| Normality of errors (Jarque Bera Test) | JB<sub>calc</sub> | 0.470459(Prob.=0.79039) |

* Working with a level of significance \( \alpha = 0.05 \), the number of exogenous variables \( k=2 \) and the number of observation \( T = 59 \), \( d_1 = 1.51 \) and \( d_2 = 1.65 \) (Durbin Watson table).

Source: the authors’ calculus

According to the new model, we can state that all parameters are significantly different from zero, at the sample level, as well as at the entire population level, a fact confirmed by the associated probabilities of accepting the null hypothesis, which are smaller than 5%. Thus, after applying the Student test, we can conclude that the null hypothesis \( H_0 \) is rejected for all parameters of the equation and the alternative hypothesis is accepted, according to which there is a link between the dependent variable and the independent variables at the level of the entire population. We also see that by eliminating the “working population” variable, the structure of the production function is not changed; the \( R^2 \) adjusted determination coefficient as well as the \( \alpha \) and \( \beta \) coefficients have similar values to those in the first equation.

In order to measure the intensity of the endogenous variable dependency from regression factors, we have found out the determination coefficient. At a sample level, between the endogenous variable – GDP, and the exogenous variable – capital stock and export – there is an average intensity link, because \( R^2 \) adjusted = 0.468424. In order to find out if this intensity is kept at the level of the entire population, we have used the Fisher test. Because \( F_{\text{calc}} = 26.55 > F_{\text{tab}} = 3.15 \), the null hypothesis according to which there is no link between the variables is rejected, meaning that the influence of the exogenous variable on the endogenous variable is significant at the level of the entire population.

As to what the testing of the fundamental hypothesis referring to the random variable \( u_t \) of the new model, we have reached at the following conclusions:

-the independence hypothesis of the values of the residual variable \( u_t \) is confirmed this time, because the Durbin-Watson statistic is equal to 1.705635, so that \( d_2 = 1.65 < DW_{\text{calc}} = 1.730679 < 4-d_2 = 2.35 \), meaning the errors of the model are independent;

-the homoscedasticity hypothesis of the residual variable \( u_t \) is confirmed, because, as the data from table 2 shows, the probability related to the Fisher statistic is higher than 5%, which determines the acceptance of the \( H_0 \) hypothesis as being true, meaning that there is a link between the residual variable, the exogenous variable and the square of the exogenous variable.
The normality hypothesis of the random variable $u_t$ is confirmed. Because of the importance of normal repartition in modeling various statistics, various special concordance tests have been built to check the normality of these various distributions. One way of checking the normality of errors hypothesis is the Jarque-Berra test, which is an asymptotic test, usable in the case of a large volume sample, which follows a chi-squared distribution with two degrees of freedom (Meșter, 2012, p: 150). Because the probability of accepting the null hypothesis as being true (Prob=0.790390) is larger than 5%, it results that the error normality hypothesis cannot be rejected at the level of the entire population, the errors being normally distributed.

Because all three hypothesis referring to the errors of the model have been checked, we can conclude that the production function that includes the export variable in the Cobb-Douglas production function is valid. Next, we want to determine the influence of the openness of the economy on the economic growth, starting again from the Cobb-Douglas production function.

3. Estimating the production function of Romania, by including the openness variable in the Cobb-Douglas production function

In order to identify the influence the openness has on the economic growth (EOP), we will estimate a new production function to which we have added this new variable.

$$ \text{GDP} = A \times K^\beta \times L^{\gamma} \times \text{EOP}^\lambda \times u_t $$  \hspace{1cm} (5)

where:

$\lambda$ represents the elasticity of the GDP based on the evolution of the openness (EOP).

By estimating the regression of the GDP variable, we have found the following results:

$$ \text{GDP} = 1.30 \times K^{1.76} \times L^{-0.85} \times \text{EOP}^{0.35} \times u_t $$  \hspace{1cm} (6)

Table 3: Estimating the regression model (5)

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C(1)</td>
<td>1.303909</td>
<td>3.407186</td>
<td>0.382694</td>
</tr>
<tr>
<td>C(2)</td>
<td>1.762346</td>
<td>0.063810</td>
<td>27.61867</td>
</tr>
<tr>
<td>C(3)</td>
<td>-0.855900</td>
<td>0.346574</td>
<td>-2.469604</td>
</tr>
<tr>
<td>C(4)</td>
<td>0.351438</td>
<td>0.206452</td>
<td>1.702274</td>
</tr>
</tbody>
</table>

R-squared: 0.948368, Mean dependent var: 10.01701, Adjusted R-squared: 0.945602, S.D. dependent var: 0.470707, S.E. of regression: 0.109785, Akaiki info criterion: -1.516251, Sum squared resid: 0.674950, Schwarz criterion: -1.376628, Log likelihood: 49.48753, Hannan-Quinn criter.: -1.461637, F-statistic: 342.8673, Durbin-Watson stat: 1.267148
As it can be seen in the table above, all parameters of the equation are significantly different from zero, at the sample level. Things are different in the case of the entire population. We can observe that the parameter of the independent EOP variable is not significantly different from zero, because the value of the Student statistic is in absolute value smaller than the table value of 2.00 for a 95% probability. Thus, we can say that the elasticity of the GDP compared to the EOP is insignificantly different from zero. This shows that the inclusion of this variable affects the significance level of the model and in order to validate it, we need to eliminate the variable.

We need to do the same for the free term too. As to what the elasticity of the potential GDP based on the working population and the capital stock is concerned, it had a value of -0.85 in the case of the working population and 1.76 in the case of the capital stock. The coefficients of the two variables are significantly different from zero at the level of the entire population.

Conclusions

According to the results we have obtained based on the two models for the production function elaborated in this study, we can conclude that the economic growth process in Romania has been influenced in a positive manner by the stock of capital and exports, while the work factor has influenced economic growth in a negative manner. Moreover, in Romania, an increase of the openness following capital imports would not generate an improvement in what type of technologies are used.

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